Intersecting Brane Worlds – A Path to the Standard Model?

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I) Introduction

Goal of superstring theory:

Embedding of the Standard Model into a unified description of gravitational and gauge forces.

Obstacles on the way:

- How to derive the precise SM spectrum?
- How to determine the precise SM couplings?
- How to break space-time supersymmetry?
- How to fix the values of the moduli?
- How to select the groundstate from an (apparent) huge vacuum degeneracy?
- How to describe the cosmological evolution of the universe?
- What is the structure of space and time at short distances?

I) Introduction

Further plan of the talk:

II) Intersecting brane world models:

- Local intersecting D-brane constructions
- The question of space-time supersymmetry
- Compactifications – embedding of intersecting branes into a compact CY-manifold

III) Phenomenological issues

- MSSM-like models and gauge coupling unification
- Proton decay (SU(5)-models)

(IV) D-brane and flux compactifications)

V) Conclusions

II) Intersecting Brane World Models

The progress in type II string physics was made possible due the discovery of D-branes.

(Polchinski)

D(p)-branes are higher(p)-dimensional topological defects, i.e. hypersurfaces, on which open strings are free to move.

They have led to several new insights:

- Non-Abelian gauge bosons as open strings on the world volumes π of the D-branes $\longrightarrow$ Brane world models

- Chiral fermions are open strings living on the intersections of two D-branes

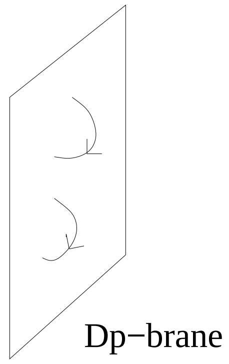
$$N_F = I_{ab} \equiv \#(\pi_a \cap \pi_b) \equiv \pi_a \circ \pi_b$$

- Correspond to non-trivial gravitational backgrounds $\longrightarrow$ AdS/CFT correspondence

II) Intersecting Brane World Models (flat branes)

Consider first flat D-branes in Minkowski space $\mathbb{R}^{1,9}$.

Simplest D-brane configuration: 1 single Dp-brane:



Massless open string spectrum: $U(1)$ gauge boson $\longrightarrow$ supersymmetric $U(1)$ gauge theory in $p+1$ dimensions

$$\mathcal{S}_{\text{eff}} = \int_\pi dx^{p+1} \; (\underbrace{\mathcal{L}_{\text{DBI}}(g, F, \phi)}_{\text{Tension}} + \underbrace{\mathcal{L}_{\text{CS}}(F, C_{p+1})}_{\text{Charge}})$$

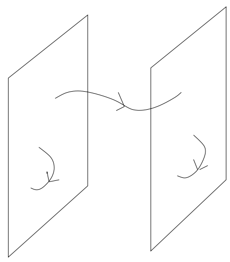
Effective gauge interactions due to the exchange of open strings:

$$\begin{aligned} S_{DBI} &= \tau_p \int \mathrm{d}^{p+1}x \sqrt{\det(g_{\mu\nu} + \tau^{-1} F_{\mu\nu})} \\ &= \left(\frac{M_{\text{string}}^{p-3}}{g_{\text{string}}}\right) \int \mathrm{d}^{p+1}x \, F_{\mu\nu}^2 + \dots \end{aligned}$$

II) Intersecting Brane World Models (flat branes)

Other D-brane configurations (in flat space-time):

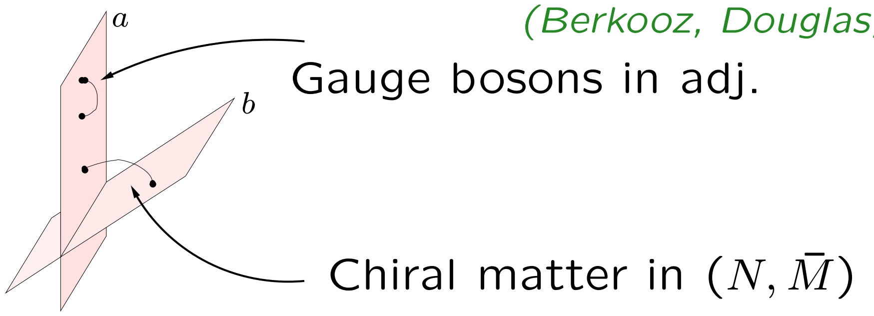
- N parallel Dp-branes



$\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory in $p + 1$ dimensions

- Intersecting D-branes

(Berkooz, Douglas, Leigh)



Open string spectrum:

(i) $\mathcal{N} = 4$ gauge bos. in adj. repr. of $U(N) \times U(M)$

(ii) Massless fermions in chiral $(N, \bar{M})$ repres.

(iii) Massive scalars in $(N, \bar{M})$ repres.

II) Intersecting Brane World Models (flat branes)

Intersecting D-branes break space-time supersymmetry!
This supersymmetry breaking manifests itself as the a massive/tachyonic scalar groundstate:

$$M_{ab}^2 = \frac{1}{2}\sum_I \Delta\Phi_{ab}^I - \max\{\Delta\Phi_{ab}^I\}$$

Massless scalars $\Leftrightarrow$ open string sector is supersymmetric.

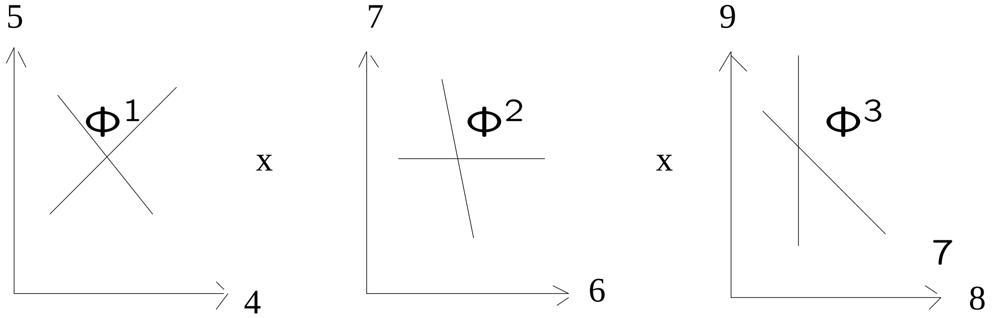
Two flat supersymmetric D6-brane configurations:

- 2 intersecting D6-branes, common in 123-directions, intersect in 4-5 and 6-7 planes, parallel in 8-9 plane:

  1/4 BPS ($\mathcal{N} = 2$ SUSY): $\Phi^1 + \Phi^2 = 0$

- 2 intersecting D6-branes, common in 123-directions, intersect in 4-5, 6-7 and 8-9 planes:

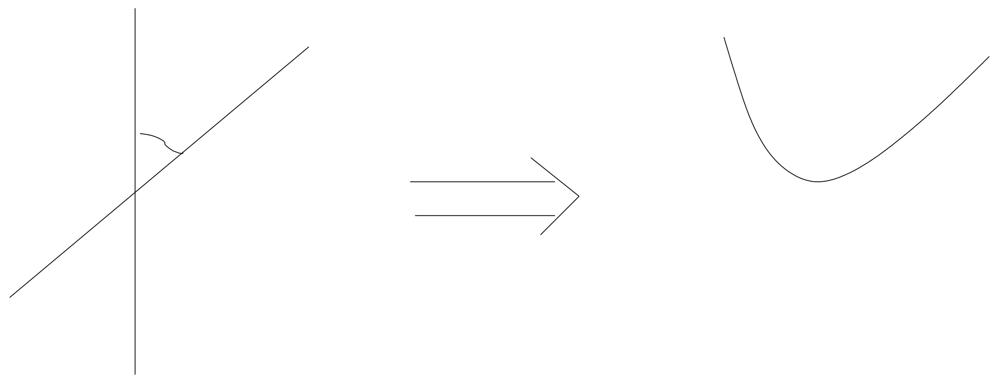
  1/8 BPS ($\mathcal{N} = 1$ SUSY): $\Phi^1 + \Phi^2 + \Phi^3 = \pi$



II) Intersecting Brane World Models (flat branes)

In case the open string scalar is tachyonic ($M_{ab}^2 < 0$) $\longrightarrow$ the 2 different branes will recombine into a single brane.
Brane recombination $\longleftrightarrow$ Tachyonic Higgs effect

(Sen)



World volume field theories (Defect field theories):

(Erdmenger, Guralnik, Helling, Kirsch, hep-th/0309043)

Higgs field arieses as impurity field.

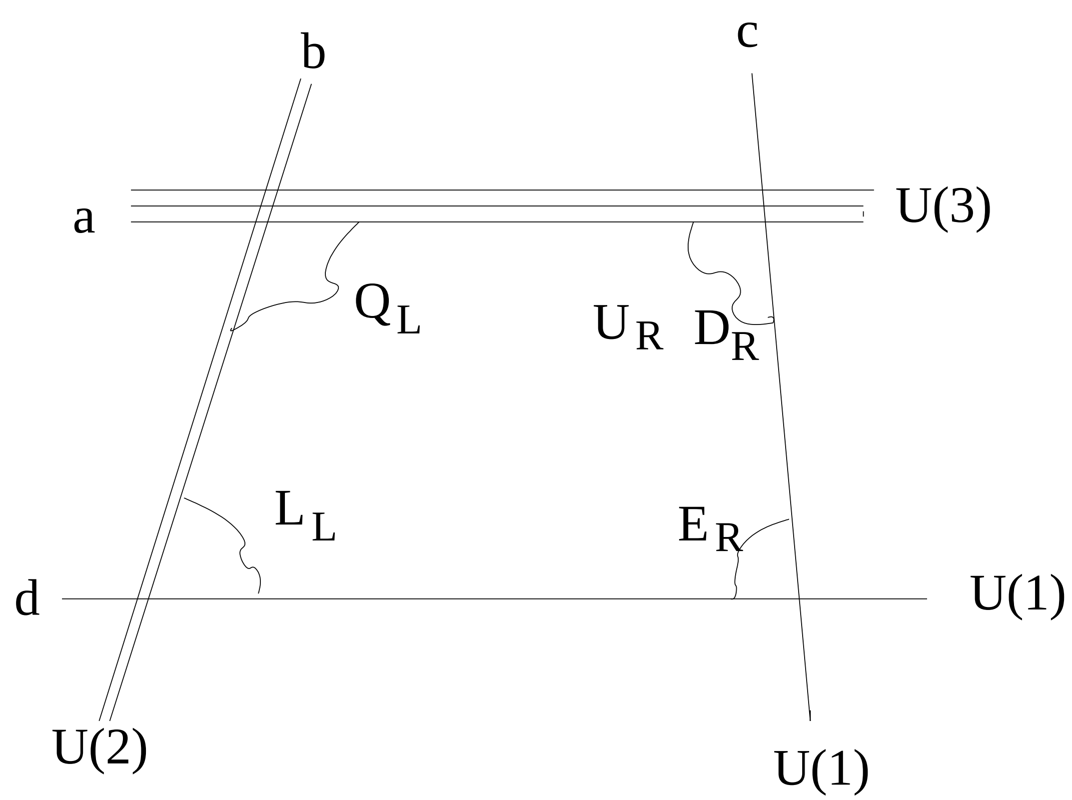
The vacuum strucure is determined by the D- and F-flatness conditions in the world volume field theory.
$\Longleftrightarrow$ Geometric calabration conditions of (recombinded) branes!

II) Intersecting Brane World Models (flat branes)

Local picture of the Standard Model:

Minimal realization by four stack of D-branes:

Stack a:	$N_a = 3$	$SU(3)_a \times U(1)_a$	QCD branes
Stack b:	$N_b = 2$	$SU(2)_b \times U(1)_b$	weak branes
Stack c:	$N_c = 1$	$U(1)_c$	right brane
Stack d:	$N_d = 1$	$U(1)_d$	leptonic brane



II) Intersecting Brane World Models (compact space)

Now we have to embed these D-branes into a compact space:

- Multiple intersections $\Longrightarrow$ Family number
- Cancellation of internal Ramond charges on compact space (Gauss law)
- Cancellation of internal brane tensions/forces (stability problem)

$\Longrightarrow$ Need orientifold plane(s) (branes of negative RR-charge and tension).

$\Longrightarrow$ Strong restrictions on brane configurations

II) Intersecting Brane World Models (compact space)

(Blumenhagen, Braun, Körs, D.L., hep-th/0206038)

(i) Choose compact type IIA orientifold background

$\mathcal{M}^{10} = (\mathbb{R}^{3,1} \times \mathcal{M}^6)/(\Omega\overline{\sigma})\,, \quad \Omega$: world sheet parity

$\overline{\sigma}$: $z_i \longrightarrow \bar{z}_i$ anti-holomorphic involution. The orientifold 6-plane is the fixed locus

$$\mathbb{R}^{3,1} \times \mathrm{Fix}(\overline{\sigma}) = \mathbb{R}^{3,1} \times \pi_{O6},$$

where $\mathrm{Fix}(\overline{\sigma})$ is a supersymmetric (sLag) 3-cycle on $\mathcal{M}^6$.

(ii) Introduce D6-branes with world-volume

$$\mathbb{R}^{3,1} \times \pi_a,$$

i.e. wrapped around the supersymmetric (sLag) 3-cycles π_a and their $\Omega\overline{\sigma}$ images π'_a of $\mathcal{M}^6$.

Massless spectrum:

- $\mathcal{N} = 1$ supergravity in the 10D bulk
- 7-dim. $\mathcal{N} = 1$ $U(N_a)$ gauge bosons localized on the D6-branes wrapped around 3-cycles π_a $(codim = 3)$.
- 4-dim. chiral fermions localized on the intersections of the D6-branes $(codim = 6)$.

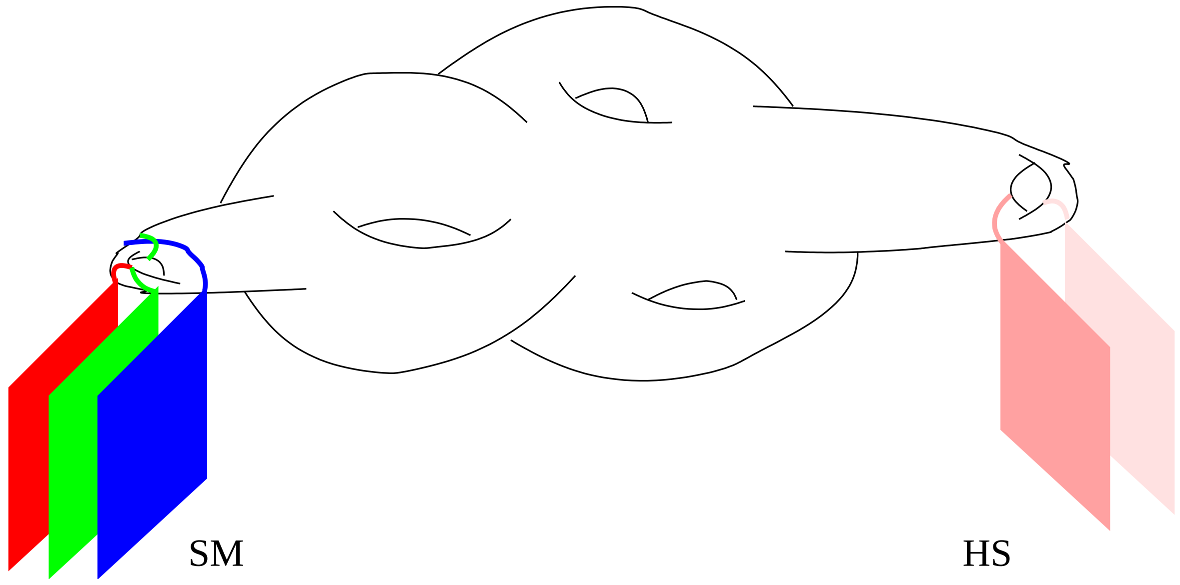
Since the chiral spectrum has to satisfy some anomaly constraints, we expect that it is given by purely topological data (Atiyah-Singer index theorem).

The chiral massless spectrum indeed is completely fixed by the topological intersection numbers of the 3-cycles of the configuration.

Sector	**Rep.**	**Number**
$a'\,a$	A_a	$\frac{1}{2}\,(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a)$
$a'\,a$	S_a	$\frac{1}{2}\,(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a)$
$a\,b$	$(\overline{N}_a, N_b)$	$\pi_a \circ \pi_b$
$a'\,b$	(N_a, N_b)	$\pi'_a \circ \pi_b$

The non-abelian gauge anomalies will cancel after satisfying the tadpole conditions and $U(1)$ anomalies are canceled by a generalized Green-Schwarz mechanism involving dimensionally reduced RR-forms.

View on the internal Calabi-Yau space:



II) Intersecting Brane World Models (compact space)

3 possibilities for supersymmetry breaking:

- SM-branes are non-supersymmetric:

$$M_{\text{susy}} \simeq M_{\text{string}} \sim \mathcal{O}(1 \text{ TeV})$$

  Need for large transversal dimensions $R_\perp$ on the CY!

- SM-branes are supersymmetric ("local" supersymmetry), but are non-supersymmetric with respect to hidden sector branes $\longrightarrow$

  Gravity mediated supersymmetry breaking:

$$M_{\text{susy}} \simeq \frac{M_{\text{string}}^2}{M_{\text{Planck}}} \simeq \mathcal{O}(1\text{TeV}) \;\Rightarrow\; M_{\text{string}} \simeq \mathcal{O}(10^{11}\text{GeV})$$

  Here the transversal dimensions on the CY are only moderately enlarged, $R_\perp \simeq \mathcal{O}(10^9)$GeV).

- All branes are supersymmetric ("global" supersymmetry) $\longrightarrow$

  Dynamical supersymmetry breaking in hidden sector:

$$M_{\text{susy}} \simeq \frac{M_{\text{hidden}}^3}{M_{\text{Planck}}^2} \simeq \mathcal{O}(1\text{TeV}) \;\Rightarrow\; M_{\text{hidden}} \simeq \mathcal{O}(10^{13}\text{GeV})$$

Consistency requirements for intersecting branes:

(i) RR-charge cancellation:

Chern-Simons actions:

$$
\begin{aligned}
\mathcal{S}_{\mathrm{CS}}^{(\mathrm{D}p)} &= \mu_p \int_{\mathrm{D}p} \mathrm{ch}(\mathcal{F}) \wedge \sqrt{\frac{\hat{\mathcal{A}}(\mathcal{R}_T)}{\hat{\mathcal{A}}(\mathcal{R}_N)}} \wedge \sum_q C_q, \\
\mathcal{S}_{\mathrm{CS}}^{(\mathrm{O}p)} &= -2^{p-4} \mu_p \int_{\mathrm{O}p} \sqrt{\frac{\hat{\mathcal{L}}(\mathcal{R}_T/4)}{\hat{\mathcal{L}}(\mathcal{R}_N/4)}} \wedge \sum_q C_q.
\end{aligned}
$$

For the case of D6-branes $\rightarrow$ equation of motion of C_7:

$$
\frac{1}{\kappa^2} d \star dC_7 = \mu_6 \sum_a N_a\, \delta(\pi_a) + \mu_6 \sum_a N_a\, \delta(\pi'_a) + \mu_6 Q_6\, \delta(\pi_{\mathrm{O6}}),
$$

Integrate over $\mathcal{M}^6 \rightarrow$ RR-tadpole cancellation as equation in homology:

$$
\sum_a N_a\,(\pi_a + \pi'_a) - 4\pi_{\mathrm{O6}} = 0.
$$

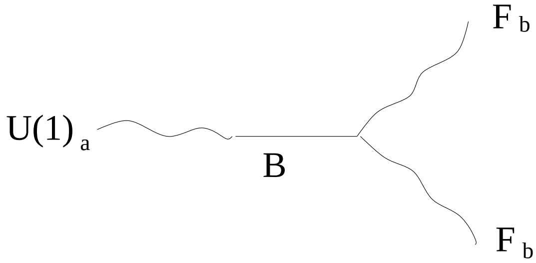
The cancellation of the RR tadpoles implies absence of the non-Abelian anomalies in the effective 4D field theory!

However there can be still anomalous $U(1)$ gauge symmetries in the effective 4D field theory.

These anomalies will be cancelled by a Green-Schwarz mechanism involving RR (pseudo)scalar fields $\Longrightarrow$ the corresponding $U(1)$ gauge boson will become massive!

Two relevant couplings in the effective action:

$$\int_{R^{3,1}\times\pi_a} C_5 \wedge \mathrm{Tr} F_a \sim \int_{R^{3,1}} B \wedge F_a \,,$$
$$\int_{R^{3,1}\times\pi_b} C_3 \wedge \mathrm{Tr}(F_b \wedge F_b) \sim \int_{R^{3,1}} \phi \wedge (F_b \wedge F_b)\,.$$



Condition for an anomaly free $U(1)_a$:

$$N_a(\pi_a - \pi_a') \circ \pi_b = 0\,.$$

2 remarks:

Even an anomaly free $U(1)$ can become massive.

The massive $U(1)$ remains as a global symmetry.

(ii) Stability of the scalar potential: NS tadpole cancellation

Due to the tension of the D-branes a vacuum energy $\mathcal{V}(\phi_4, U_i)$ is induced which depends on the NS background fields: dilaton ϕ_4 , complex structure moduli U_i.

For flat 4-dim. Minkowski space-time we need a stable minimum of $\mathcal{V}(\phi_4, U_i)$ with $\mathcal{V}_{\rm min} = 0 \longleftrightarrow$ Vanishing of NS tadpoles!

Scalar (D-term) potential:

$$
\begin{aligned}
\mathcal{V} &= \tau_6 \frac{e^{-\phi_4}}{\sqrt{\mathrm{Vol}(\mathcal{M}^6)}} \left(\sum_a N_a \ \mathrm{Vol}(\mathrm{D6}_a) - 4\mathrm{Vol}(\mathrm{O6}) \right) \\
&= \tau_6 \, e^{-\phi_4} \left(\sum_a N_a \int_{\pi_a + \pi'_a} \Re(e^{i\phi_a} \widehat{\Omega}_3) - 4 \int_{\pi_{\mathrm{O6}}} \Re(\widehat{\Omega}_3) \right)
\end{aligned}
$$

Minimization of $\mathcal{V}$ will fix (part of) the complex structure moduli U_i.

3 possible scenarios:

- "Global" $\mathcal{N}=1$ supersymmetry:
  Minima are such that all angles are supersymmetric $\longleftrightarrow$ all D6-branes conserve the same supersymmetries as orientifold plane, i.e. all D6-branes be calibrated with respect to $\Re(\widehat{\Omega}_3)$ $\Rightarrow \mathcal{V}_{\text{min}} = 0$

- "Local" $\mathcal{N}=1$ supersymmetry:
  Minima are such that only SM angles are supersymmetric $\longleftrightarrow$ only SM D6-branes conserve the same supersymmetries as orientifold plane, i.e. only SM D6-branes be calibrated with respect to $\Re(\widehat{\Omega}_3)$. (Here hidden sector is in general necessary for RR tadpole cancellation.)

- No supersymmetry:
  Minima are such that SM angles are non-supersymmetric $\longleftrightarrow$ SM D6-branes do not conserve the same supersymmetries as orientifold plane. (Stability is very difficult to achieve!)

II) Intersecting Brane World Models (compact space)

Model building: Two simple ways to embed the SM!
(Blumenhagen, Körs, D.L., hep-th/0012156; Cremades, Ibanez, Marchesano, Rabadan, hep-th/0105155, hep-th/0302105)

Both of them use four stacks of $D6$-branes:

$$\begin{aligned} A &: \quad U(3)_a \times SP(2)_b \times U(1)_c \times U(1)_d \\ B &: \quad U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d. \end{aligned}$$

The chiral spectrum of the intersecting brane world model should be identical to the chiral spectrum of the standard model particles.

This fixes uniquely the intersection numbers of the four 3-cycles, $(\pi_a, \pi_b, \pi_c, \pi_d)$.

field	**sector**	**I**	$SU(3) \times SU(2) \times U(1)^3$
q_L	(ab)	3	$(3, 2; 1, 0, 0)$
u_R	(ac)	3	$(\overline{3}, 1; -1, 1, 0)$
d_R	(ac')	3	$(\overline{3}, 1; -1, -1, 0)$
e_L	(db)	3	$(1, 2; 0, 0, 1)$
e_R	(dc')	3	$(1, 1; 0, -1, -1)$
ν_R	(dc)	3	$(1, 1; 0, 1, -1)$

The hypercharge Q_Y is given as the following linear combination of the three $U(1)$s

$$Q_Y = \frac{1}{3}Q_a - Q_c - Q_d.$$

Then an ISB model is constructed by the following six steps:

(i) chose a compact Calabi-Yau manifold $\mathcal{M}_6$,

(ii) determine the orientifold 6-plane π_{O6}

(iii) chose four 3-cycles $\pi_{U(3)_a}$, $\pi_{U(2)_b}$, $\pi_{U(1)_c}$, $\pi_{U(1)_d}$ for the four stacks of D6-branes, as well as their orientifold mirrors

(iv) compute their intersection numbers

(v) ensure that the RR tadpole conditions vanish (possibly by adding hidden D6-branes)

(vi) ensure that the linear combination $U(1)_Y$ remains massless.

$$\sum_i c_i\, N_i\, \left(\pi_i - \pi_i'\right) = 0.$$

II) Intersecting Brane Worlds (compact spaces)

Many non-supersymmetric as well as $\mathcal{N} = 1$ supersymmetric intersecting brane world models on tori, orbifolds, or the quintic Calabi-Yau manifold with gauge group

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y$$

and 3 families of quark and leptons can be explicitly constructed.

(Blumenhagen, Braun, Görlich, Ott, Körs, D.L. (2000/01/02); Aldazabal, Cremades, Franco, Ibanez, Marchesano, Rabadan, Uranga; Cvetic, Shiu, Uranga; Bailin, Kraniotis, Love; Kokorelis; Förste, Honecker, Schreyer; Ellis, Kanti, Nanopoulos)

(Some of the authors also use more than four stack of D6-branes or different types of D-branes.)

Example: **Fermat quintic CY_3:**

$$P(z_i) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0 \subset \mathbb{CP}^4$$

- O6-plane: sLag $\mathbb{RP}^3$, $\bar{\sigma}$-fixed set $\pi_{O6} = \pi_{0,0,0,0}$:

  $$P(x_i) = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0 \subset \mathbb{RP}^4$$

- D6-branes: Use $\mathbb{Z}_5^4$, $z_i \mapsto \omega^{k_i} z_i$, $\omega = e^{2\pi i/5}$, $k_i \in \mathbb{Z}_5$ $\longrightarrow$ sLag 3-cycle $\pi_a = \pi_{k_2,k_3,k_4,k_5}$:

  $$x_1^5 + \Re(\omega^{k_2} z_2)^5 + \Re(\omega^{k_3} z_3)^5 + \Re(\omega^{k_4} z_4)^5 + \Re(\omega^{k_5} z_5)^5 = 0$$

  $5^4 = 625$ sLag $\mathbb{RP}^3$'s, calibrated with $\Re(\prod_i \omega^{k_i} \Omega_3)$. (125 sLag's are $\Re(\Omega_3)$ calibrated $\longrightarrow \mathcal{N} = 1$ SUSY)

- Intersection numbers of π_{k_2,k_3,k_4,k_5} and π_{O6} from *Brunner, Douglas, Lawrence, Römelsberger, hep-th/9906200*

  (The 125 SUSY D6-branes have zero intersection #'s $\longrightarrow$ no $\mathcal{N} = 1$ chiral models from the quintic!)

Non-supersymmetric Standard Model from the quintic:

- Introduce four stacks of D6-branes with $N_a = 3$, $N_b = 2$ and $N_c = N_d = 1$ $\longrightarrow$ gauge group:

$$G = U(3) \times U(2) \times U(1)^2$$

- Choose the following "wrapping numbers":

$$\begin{aligned}
\pi_a &= \pi_c - \pi_d - |0,2,1,4\rangle - |0,3,4,1\rangle, \\
\pi_b &= |0,3,1,1\rangle, \\
\pi_c &= |1,4,3,4\rangle + |4,4,3,2\rangle, \\
\pi_d &= |0,3,0,3\rangle - |2,0,3,4\rangle
\end{aligned}$$

  This produces the intersection numbers of the standard model with

  3 generation of quarks and leptons

- Anomaly-free, massless hypercharge is

$$U(1)_Y = \frac{1}{3} U(1)_a - U(1)_c + U(1)_d$$

- GS couplings to cancel $U(1) - SU(N)^2$ anomalies.

- An hidden sector needed for tadpole cancellation.

III) Phenomenology (gauge coupling unification)

(Blumenhagen, Stieberger, D.L., hep-th/0305146; cfr. Antoniadis, Kiritsis, Tomaras, hep-th/0004214)

The three Standard Model gauge couplings g_s, g_w and g_y have different values at the weak scale.

Extrapolating these couplings due to the one-loop running

$$\frac{4\pi}{g_a^2(\mu)} = k_a \frac{4\pi}{g_X^2} + \frac{b_a}{2\pi} \log\left(\frac{\mu}{M_X}\right) + \Delta_a$$

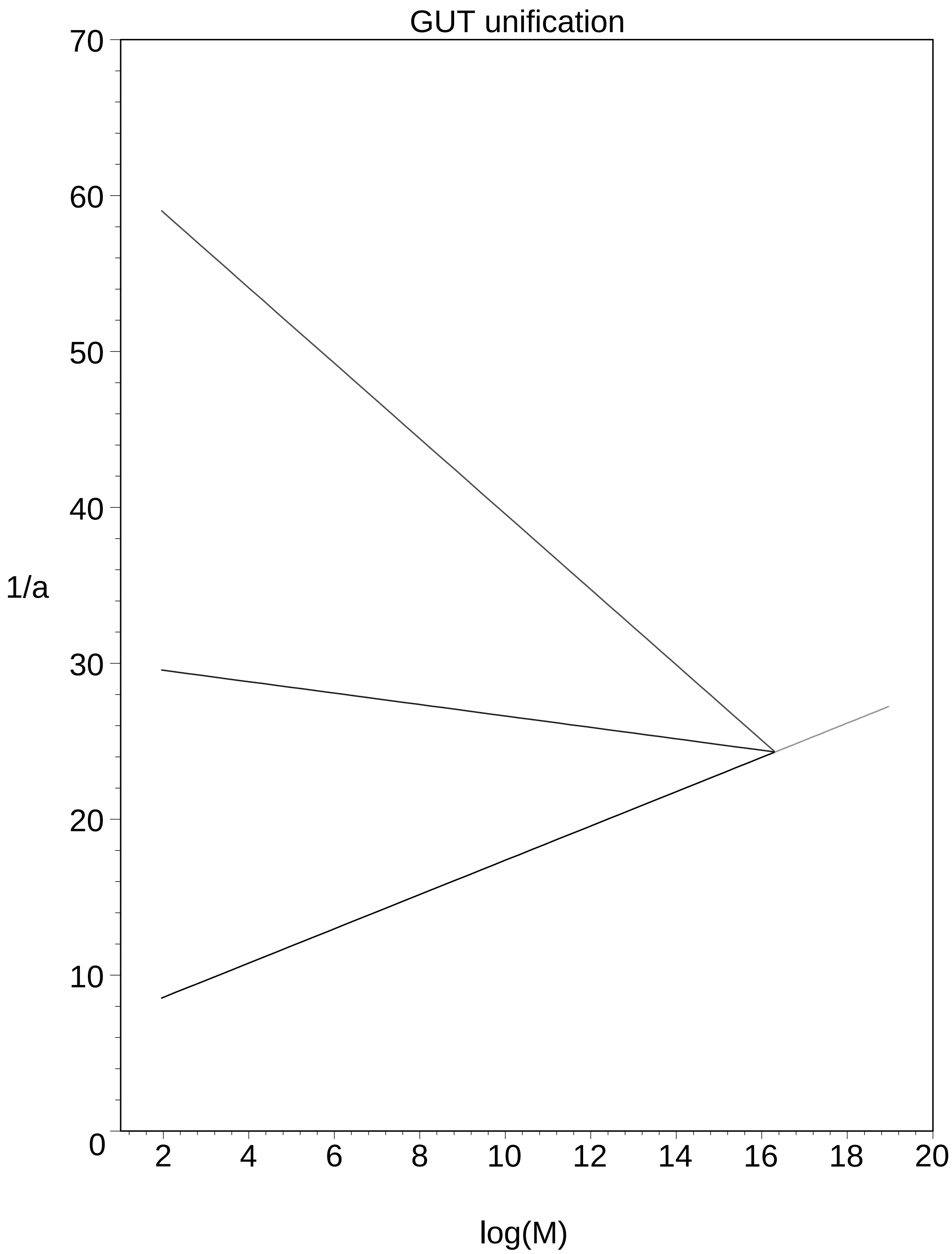
to higher scales, one finds that they all meet at

$$M_X \simeq 2 \cdot 10^{16} \text{ GeV}, \qquad \alpha_s = \alpha_w = \frac{3}{5}\alpha_Y = \alpha_X \simeq \frac{1}{24},$$

if the light spectrum contains just the MSSM particles.

This is in accord with for instance an $SU(5)$ Grand Unified gauge group at the GUT scale.

III) Phenomenology (gauge coupling unification)



III) Phenomenology (gauge coupling unification)

In string theory one has a new scale M_s, so that it is natural to relate M_X to M_s. In the heterotic string one finds

$$k_a = \text{level of SU(N}_\text{a}\text{) Kac} - \text{Moody algebra.}$$

At one loop level the relation between the string and the Planck scale was found to be

$$M_s \simeq g_{st} \cdot 0.058 \cdot M_{pl},$$

which, using $g_{st} \simeq 0.7$, led to $M_s \simeq 5 \cdot 10^{17}$ GeV.

(Kaplunovsky (1988); Derendinger, Ferrara, Kounnas, Zwirner (1992))

The discrepancy between M_X and M_s needs to be explained by moduli-dependent string threshold corrections Δ_a (or alternatively by heterotic M-theory).

(Ibanez, Ross, D.L. (1991/92); Nilles, Stieberger; Witten (1996) ...)

In D-brane models with gauge group $SU(3) \times SU(2) \times U(1)_Y$ each gauge factor comes with its own gauge coupling, which at string tree-level can be deduced from the Dirac-Born-Infeld action

$$\frac{4\pi}{g_a^2} = \frac{M_s^3\, V_a}{(2\pi)^3\, g_{st}\, \kappa_a}, \qquad V_a = (2\pi)^3 R_a^3 .$$

with $\kappa_a = 1$ for $U(N_a)$ and $\kappa_a = 2$ for $SP(2N_a)/SO(2N_a)$.

By dimensionally reducing the type IIA gravitational action one can similarly express the Planck mass in terms of stringy parameters ($M_{pl} = (G_N)^{-\frac{1}{2}}$)

$$M_{pl}^2 = \frac{8\, M_s^8\, V_6}{(2\pi)^6\, g_{st}^2}, \qquad V_6 = (2\pi)^6 R^6 .$$

Eliminating the unknown string coupling g_{st} gives

$$\frac{1}{\alpha_a} = \frac{M_{pl}}{2\sqrt{2}\, \kappa_a\, M_s} \frac{V_a}{\sqrt{V_6}}.$$

Due to

$$\frac{V_a}{\sqrt{V_6}} = \int_{\pi_a} \Re(e^{i\phi_a} \widehat{\Omega}_3)$$

the gauge coupling only depends on the complex structure moduli.

III) Phenomenology (gauge coupling unification)

Consider the gauge coupling unification in a model indepedent bottom up approach.

3 phenomenological requirements:

- The SM branes mutually preserve $\mathcal{N} = 1$ supersymmetry.
- The intersecting numbers realize a 3 generation MSSM
- The $U(1)_Y$ gauge boson is massless

One can show that these restrictions provide one relation between the 3 gauge coupling constants:

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}.$$

This relation will allow for natural gauge coupling unification!

In the absence of threshold corrections, the one-loop running of the three gauge couplings is described by the well known formulas

$$\begin{aligned}\frac{1}{\alpha_s(\mu)} &= \frac{1}{\alpha_s} + \frac{b_3}{2\pi}\ln\left(\frac{\mu}{M_s}\right)\\ \frac{\sin^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_w} + \frac{b_2}{2\pi}\ln\left(\frac{\mu}{M_s}\right)\\ \frac{\cos^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_Y} + \frac{b_1}{2\pi}\ln\left(\frac{\mu}{M_s}\right),\end{aligned}$$

where (b_3, b_2, b_1) are the one-loop beta-function coefficients for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$.

Using the tree level relation at the string scale yields

$$\frac{2}{3}\frac{1}{\alpha_s(\mu)} + \frac{2\sin^2\theta_w(\mu) - 1}{\alpha(\mu)} = \frac{B}{2\pi}\ln\left(\frac{\mu}{M_s}\right)$$

with

$$B = \frac{2}{3}b_3 + b_2 - b_1.$$

III) Phenomenology (gauge coupling unification)

Employing the measured Standard Model parameters

$$\begin{aligned} M_Z &= 91.1876 \text{ GeV}, \quad \alpha_s(M_Z) = 0.1172, \\ \alpha(M_Z) &= \frac{1}{127.934}, \quad \sin^2\theta_w(M_Z) = 0.23113 \end{aligned}$$

the resulting value of the unification scale only depends on the combination B of the beta-function coefficients.

For the MSSM one has $(b_3, b_2, b_1) = (3, -1, -11)$, i.e $B = 12$ and the unification scale is the usual GUT scale

$$M_s = M_X = 2.04 \cdot 10^{16} \text{ GeV}.$$

For the individual gauge couplings at the string scale we get

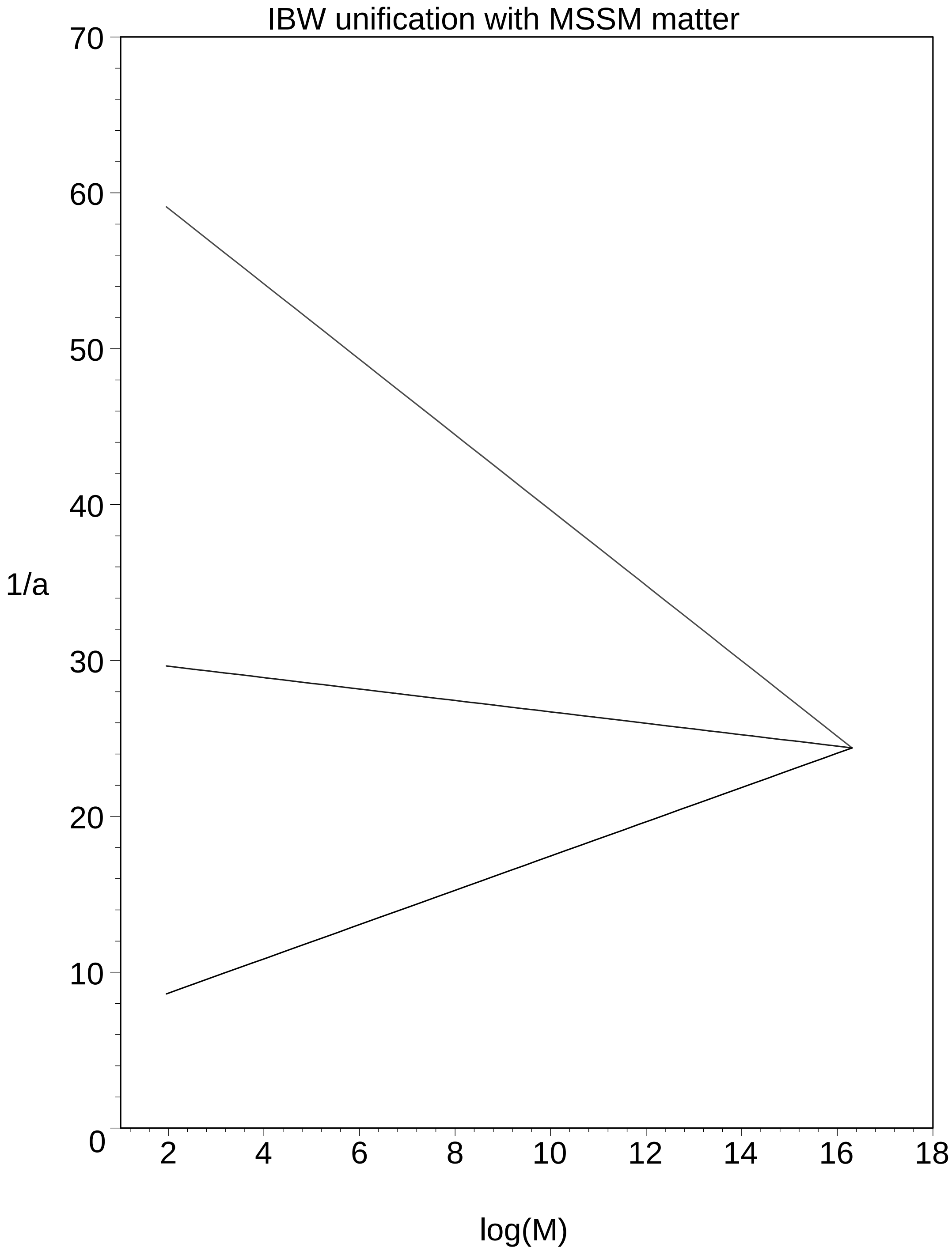
$$\alpha_s(M_s) = \alpha_w(M_s) = \frac{5}{3}\alpha_Y(M_s) = 0.041,$$

which are just the supersymmetric GUT scale values with the Weinberg angle being $\sin^2\theta_w(M_s) = 3/8$.

Assuming $g_{st} = g_X$, one obtains for the overall radius R and the internal radii R_s, R_w

$$M_s R = 5.32, \quad M_s R_s = 2.6, \quad M_s R_w = 3.3.$$

III) Phenomenology (gauge coupling unification)



In general besides the chiral matter string theory contains also additional vector-like matter.

This is also localized on the intersection loci of the $D6$ branes and also comes with multiplicity n_{ij} with $i, j \in \{a, b, c, d\}$.

One finds the following contribution to B

$$\begin{aligned} B &= 12 - 2\, n_{aa} - 4\, n_{ab} + 2\, n_{a'c} + 2\, n_{a'd} - 2\, n_{bb} + 2\, n_{c'c} \\ &\quad +2\, n_{c'd} + 2\, n_{d'd}. \end{aligned}$$

B does not depend on the number of weak Higgs multiplets n_{bc}.

Example A:

If we have a model with a second weak Higgs field, i.e. $n_{bc} = 1$, we still get $B = 12$ but with

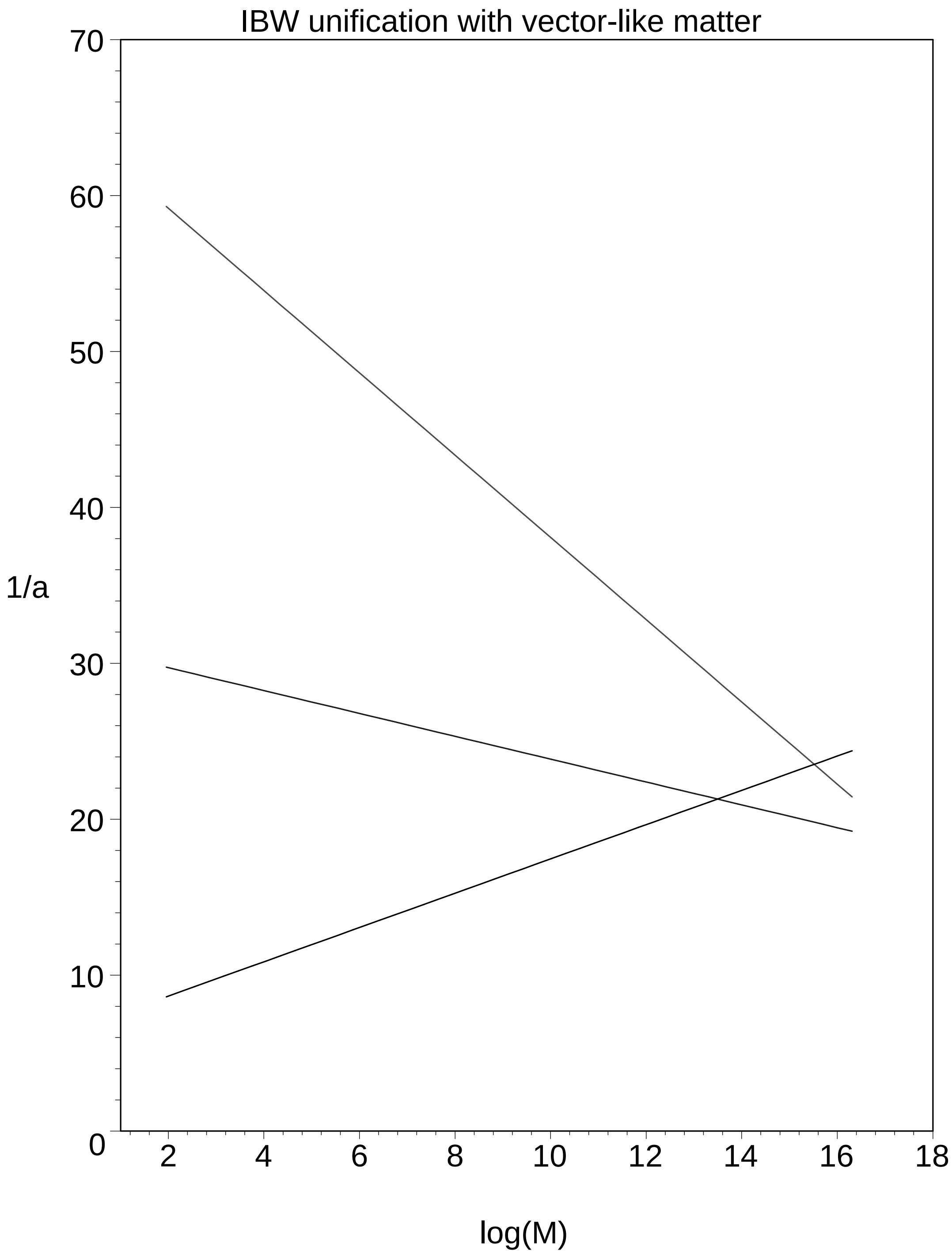
$$(b_3, b_2, b_1) = (3, -2, -12).$$

The gauge couplings "unify" at the scale

$$M_s = 2.02 \cdot 10^{16} \text{GeV}.$$

However they are not all equal at that scale

$$\alpha_s(M_s) = 0.041, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.028.$$



IBW unification with vector-like matter
1/a
70
60
50
40
30
20
10
0
2
4
6
8
10
12
14
16
18
log(M)

Example B: intermediate scale model

For models with gravity mediated supersymmetry breaking (hidden anti-branes) the string scale is naturally in the intermediate regime $M_s \simeq 10^{11}$GeV.

Choosing vector-like matter

$$n_{a'a} = n_{a'd} = n_{d'd} = 2, \quad n_{bb} = 1$$

leads to $B = 18$.

The string scale turns out to be

$$M_s = 3.36 \cdot 10^{11}\text{GeV}.$$

The running of the couplings with

$$(b_3, b_2, b_1) = (-1, -3, -65/3)$$

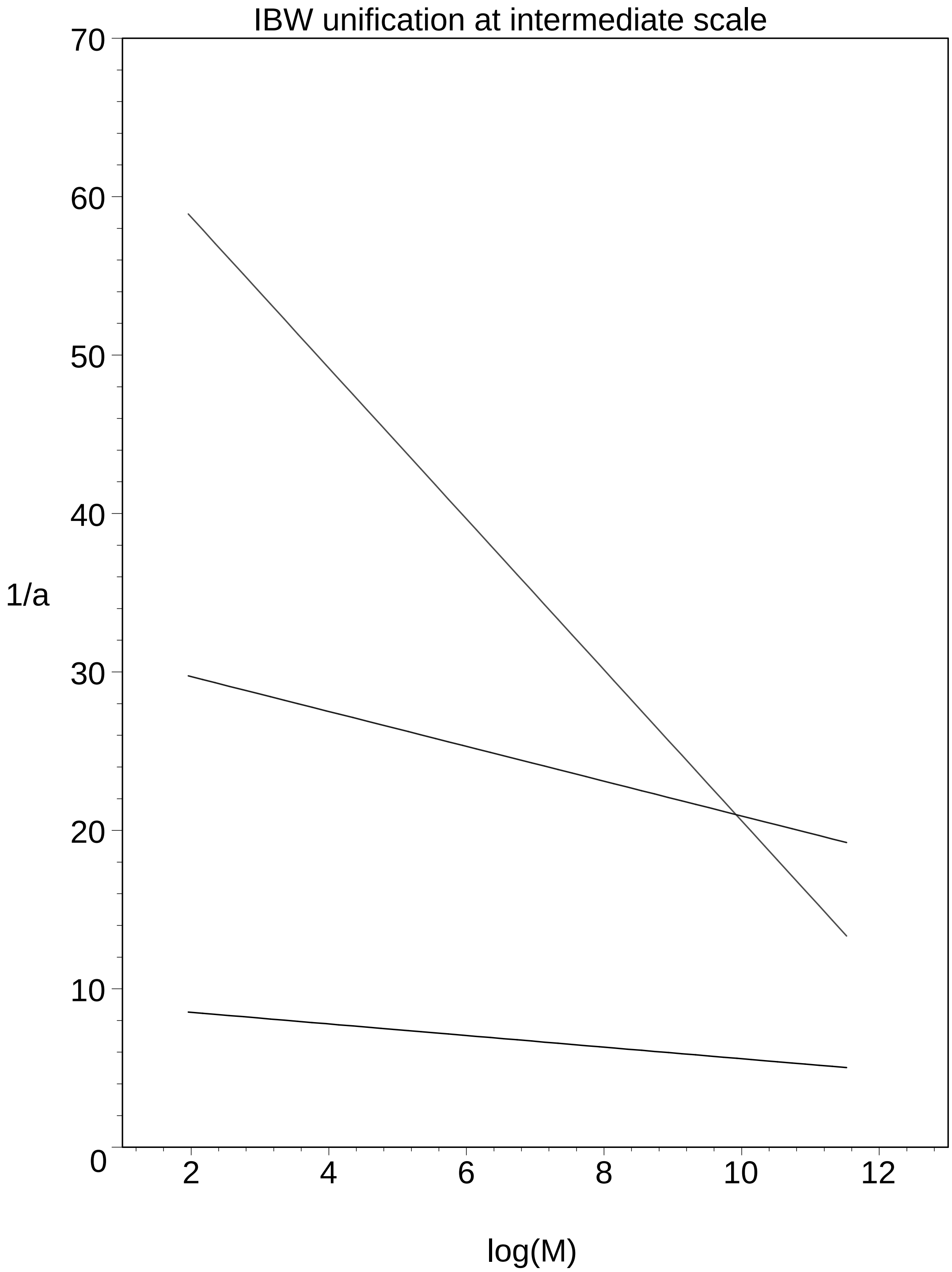
leads to the values of the gauge couplings at the string scale

$$\alpha_s(M_s) = 0.199, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.045.$$

Assuming $g_{st} \simeq 1$, one obtains for the radii

$$M_sR = 230, \quad M_sR_s = 1.7, \quad M_sR_w = 3.3.$$

III) Phenomenology (gauge coupling unification)



Example C: Planck scale model

Interestingly for $B = 10$ one gets

$$\frac{M_s}{M_{pl}} = 1.24 \sim \sqrt{\frac{\pi}{2}}.$$

Choosing vector-like matter

$$n_{aa} = 1,$$

the beta-function coefficients read

$$(b_3, b_2, b_1) = (0, -1, -11).$$

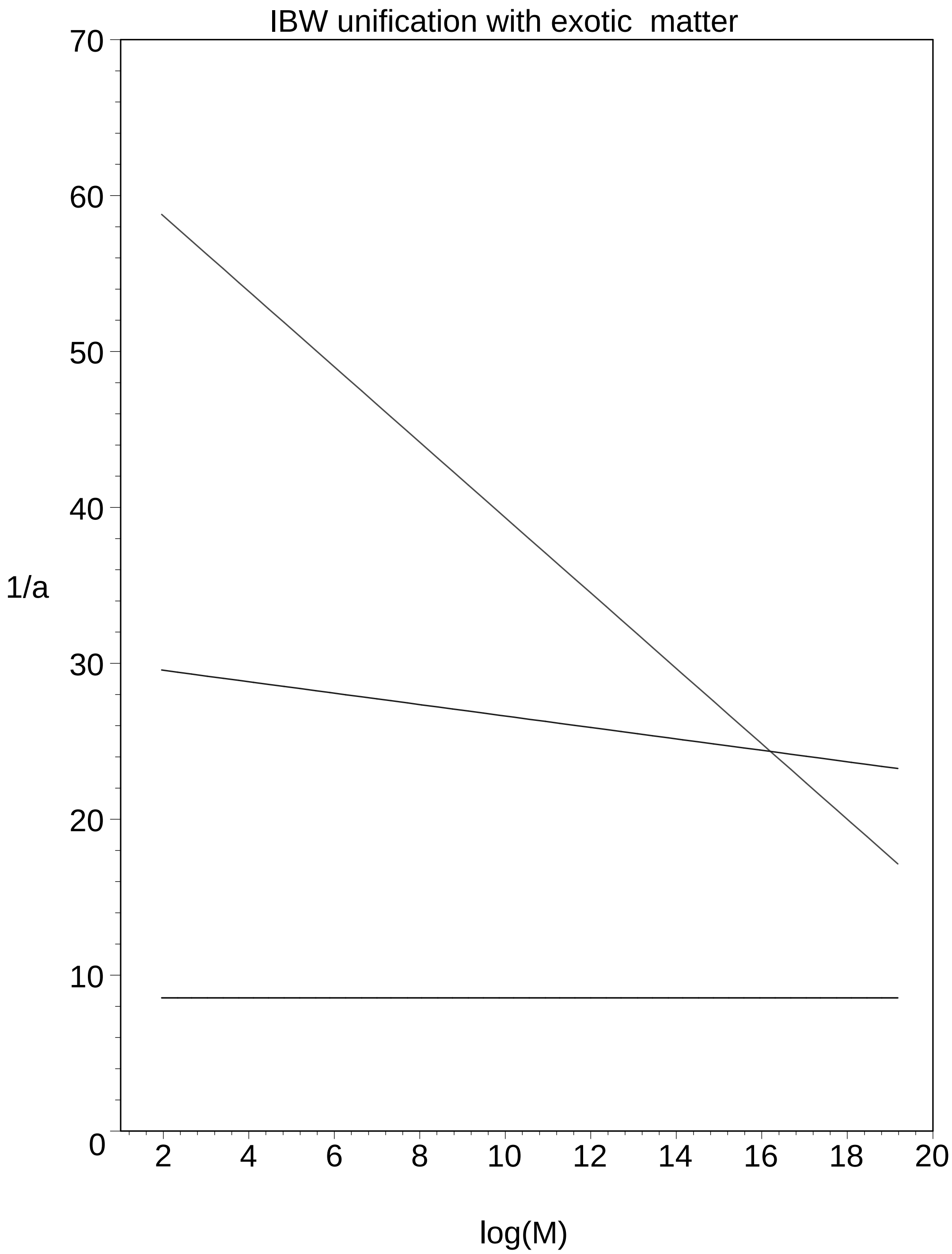
The couplings at the string scale turn out to be

$$\alpha_s(M_s) = 0.117, \quad \alpha_w(M_s) = 0.043, \quad \alpha_Y(M_s) = 0.035$$

leading to $\sin^2\theta_w(M_s) = 0.445$.

For the scales of the overall Calabi-Yau volume and the 3-cycles we obtain

$$M_s R = 0.6, \quad M_s R_s = 1.9, \quad M_s R_w = 3.3.$$


IBW unification with exotic matter
1/a
log(M)
0
10
20
30
40
50
60
70
2
4
6
8
10
12
14
16
18
20

Explicit computation of Δ_a in toroidal and orbifold models:

(S. Stieberger, D.L., hep-th/0302221.)

(i) $\mathcal{N}=4$ sectors: $\Delta_a = 0$.

(ii) $\mathcal{N}=2$ sectors:

$$\Delta_{ab}^{\mathrm{N}=2} = b_{ab}^{N=2} \; \ln(T_2^i V_a^i |\eta(T^i)|^4) + \text{const.} \; ,$$

with the wrapped brane volume

$$V_a^i = \frac{1}{U_2^i} |n_a^i + U^i m_a^i|^2 \, .$$

(iii) $\mathcal{N}=1$ sectors:

$$\Delta_{ab}^{\mathrm{N}=1} = b_{ab}^{N=1} \; \ln \frac{\Gamma(1-\frac{1}{\pi}\phi_{ab}^1)\;\Gamma(1-\frac{1}{\pi}\phi_{ab}^2)\;\Gamma(1+\frac{1}{\pi}\phi_{ab}^1+\frac{1}{\pi}\phi_{ab}^2)}{\Gamma(1+\frac{1}{\pi}\phi_{ab}^1)\;\Gamma(1+\frac{1}{\pi}\phi_{ab}^2)\;\Gamma(1-\frac{1}{\pi}\phi_{ab}^1-\frac{1}{\pi}\phi_{ab}^2)} \, ,$$

$$\cot(\phi_{ab}^j) = \frac{n_a^j n_b^j \frac{R_1^j}{R_2^j} + m_a^j m_b^j \frac{R_2^j}{R_1^j}}{n_a^j m_b^j - n_b^j m_a^j} \, .$$

(Δ_a still depend on moduli! (Cfr. heterotic $\mathcal{N}=1$ sectors: DKL))

(iv) $\mathcal{N}=0$ sectors: UV divergent 1-loop threshold corrections. Note: in “local” supersymmetric models all SM thresholds can be made finite!

(S. Stieberger, D.L.: work in progress)

III) Phenomenology (proton decay)

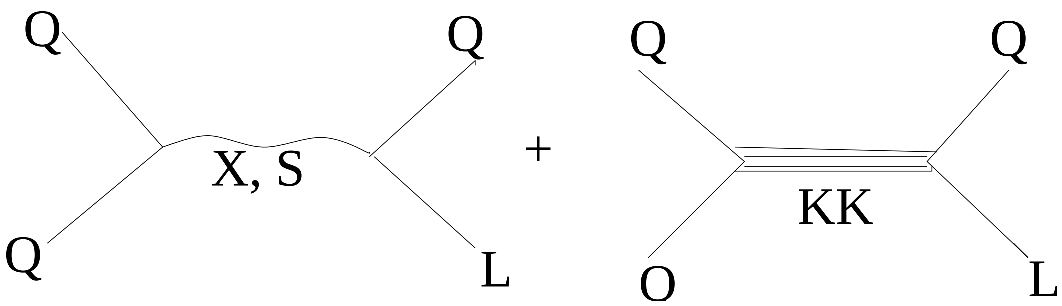
Supersymmetric GUT field theories:

Proton decay can occur due to dimension 5 operators $\int d^2\theta Q^3 L$, i.e. exchange of SUSY particles S,

or can occur due to dimension 6 operators $\int d^4\theta Q^2 \tilde{Q}^* \tilde{L}^*$, i.e. exchange of heavy gauge vector bosons X:

String theory (M-theory):

In addition, exchange of infinite tower of KK states:



Enhancement of the proton decay amplitude by an universal factor $\alpha_{GUT}^{-1/3}$!

However to know the precise numerical coefficient one has to make a model dependent calculation:

(i) M-theory on a G_2-manifold.
(T. Friedmann, E. Witten, hep-th/0211269)

(ii) Intersecting D6-brane models with $G = SU(3) \times SU(2) \times U(1)_Y$: Baryon number is an anomalous $U(1)$ gauge symmetry $\Rightarrow$ global $U(1)_B$ symmetry

$\Longrightarrow$ Proton is stable!

(iii) Intersecting D6-brane SU(5)-GUT model:
(I. Klebanov, E. Witten, hep-th/0304079)

Consider a stack of 5 D6-branes plus their orientifold mirrors D6':

(i) SU(5) gauge bosons
(ii) $10+\overline{10}$ matter from open strings at the intersection of D6,D6'-branes.

(Problem: no fundamental matter in 5-representation!)

Compute the open string disc amplitude $10^2\overline{10}^2$ ($p \to \pi^0 e_L^+$):

$$A_{st} = \pi\alpha' g_s I(\Phi_1, \Phi_2, \Phi_3) = \frac{\alpha_{GUT}^{2/3} L^{2/3} g_s^{1/3} I(\Phi_1, \Phi_2, \Phi_3)}{4\pi M_{GUT}^2},$$

$$I = \int_0^1 \frac{dx}{x(x-1)} \prod_{i=1}^{3} \sqrt{\sin(\pi\Phi_i)}[F(\Phi_i, 1-\Phi_i; 1; x) F(\Phi_i, 1-\Phi_i; 1; 1-x)]^{-1/2} \simeq 7 - 11\,.$$

III) Phenomenology (proton decay)

Compare field theory proton lifetime with string theory lifetime:

Field theory:

$$\tau_p = 1.6 \times 10^{36}\text{y}\Big(\frac{0.4}{\alpha_{GUT}}\Big)^2\Big(\frac{M_X}{2\times 10^{16}\text{GeV}}\Big)^4 \simeq 1.6\times 10^{36}\text{y}\,.$$

String theory:

$$\tau_{p,s} = (0.037\ L^{2/3} I g_s^{1/3})^{-2}\tau_p\,.$$

Two enhancement factors:

(i) Contribution of KK-states: 1-loop threshold factor $L \simeq 8$ (from M-theory on a G_2-manifold).

(ii) Tree level string disc amplitude: $I \simeq 7 - 11$.

$\Longrightarrow$ No substantial enhancement of proton lifetime!

(Problem: Dimension 5 operators are so far neglected.)

IV) Type II Compactifications with D-branes and H-fluxes

Problem of moduli stabilization:

Type IIA: D6-branes wrapped around 3-cycles $\mathcal{C}_3 \subset \mathcal{M}^6$, Potential $\mathcal{V}_{brane} \sim \mathrm{Vol}(\mathcal{C}_3) \longrightarrow$ fixes complex structure moduli U_i!

Q: How to fix the Kähler moduli $T_i \sim \mathrm{Vol}(\mathcal{C}_2)$ of $\mathcal{M}^6$?

A: Turn on H-fluxes, i.e. background expectation values for H-field strength fields!

E.g. RR 2-form field strength:

$$\langle H_R^{(2)} \rangle = \oint_{\mathcal{C}_2} H_R^{(2)} \Rightarrow \mathcal{V}_{flux} \sim \langle H_R^{(2)} \rangle^2$$

Aim: Construct compactifications with D-branes and fluxes:

(Blumenhagen, Taylor, D.L., hep-th/0303016; Cascales, Uranga, hep-th/0303024)

D6-branes: Non-Abelian gauge bosons, chiral fermions $\rightarrow$ SM, non-trivial $\mathcal{V}_{brane}(U_i)$.

H-fluxes: No chirality, non-trivial $\mathcal{V}_{flux}(T_i)$.

IV) Type II Compactifications with D-branes and H-fluxes

T-dual type IIB (mirror) picture:

D-branes: Stacks of $D9_a$ branes which wrap mirror $\tilde{\mathcal{M}}^6$ CY plus open string magnetic fields F_{ab} through 2-cycles of $\tilde{\mathcal{M}}^6 \longrightarrow$ Non-Abelian gauge bosons, chiral fermions $\rightarrow \mathcal{V}_{brane}(T_i)$, fixes Kähler moduli of $\tilde{\mathcal{M}}^6$.

H-fluxes: RR and NS 3-form flux $\langle H_R^{(3)} \rangle, \langle H_{NS}^{(3)} \rangle \neq 0$ through 3-cycles of $\tilde{\mathcal{M}}^6 \longrightarrow \mathcal{V}_{flux}(U_i)$, fixes complex structure moduli of $\tilde{\mathcal{M}}^6$.

Effective flux induced action:

(Taylor, Vafa; Kachru, Schulz, Trivedi; ...)

(i) Kinetic energy of 3-forms $\Longrightarrow$ scalar potential $\mathcal{V}_{flux}$

$$\begin{aligned} \mathcal{S}_{eff} &= -\frac{1}{4\kappa_{10}^2 \Im(\tau)} \int_{\tilde{\mathcal{M}}^6} G \wedge \star G \,, \\ G &= \tau H_{NS}^{(3)} + H_R^{(3)} \,, \quad \tau = C_0 + i e^{-\phi} \,. \end{aligned}$$

Expand G in terms of a basis of $H^3(\tilde{\mathcal{M}}^6, Z)$:

$$\begin{aligned} G &= e_\Lambda X^\Lambda + m^\Lambda F_\Lambda \,, \\ e_\Lambda &= \tau e_\Lambda^1 + e_\Lambda^2 \,, \quad m^\Lambda = \tau m_1^\Lambda + m_2^\Lambda \,. \end{aligned}$$

IV) Type II Compactifications with D-branes and H-fluxes

Scalar potential:

$$\begin{aligned}\mathcal{V}_{flux} &= -\frac{\mu_3}{2\Im\tau}[(e+m\bar{\mathcal{N}})(\Im\mathcal{N})^{-1}(\bar{e}+\bar{m}\mathcal{N})] \\ &+ \mu_3(m\times e) = \mathcal{V}_F + \mathcal{V}_D\end{aligned}$$

$\mathcal{N}$ denotes the period matrix:

$$\begin{aligned}\mathcal{N}_{\Lambda\Sigma} &= \bar{F}_{\Lambda\Sigma} + 2i\frac{\Im(F_{\Lambda\Gamma})\Im(F_{\Sigma\Delta})X^\Gamma X^\Delta}{\Im(F_{\Gamma\Delta})X^\Gamma X^\Delta}, \\ X^\Lambda &= \int_{A^\Lambda}\Omega_3\,, \quad F_\Lambda = \int_{B_\Lambda}\Omega_3\end{aligned}$$

$\mathcal{V}_{flux}$ depends on the compex structure moduli U_i and τ.

$\mathcal{V}_F$ can be derived from a superpotential:

$$W = \frac{1}{\sqrt{2}\kappa_{10}}\int_{\tilde{\mathcal{M}}^6}\Omega_3\wedge G = \sqrt{\mu}_3(e_\Lambda X^\Lambda + m^\Lambda F_\Lambda)$$

For certain choices of fluxes with $N_{flux} = m\times e \neq 0$ supersymmetric minimima of W with $W_{U_i} = W_\tau = W = 0$, i.e. $\mathcal{V}_F = 0$ can be found.

Note: Since at the minimum $V_{flux} = \mu_3(m\times e) > 0$ one needs orientifold planes to cancel the vacuum energy of the 3-form fluxes.

(ii) Topological action of 3-forms $\Longrightarrow$ RR-tadpoles

$$\mathcal{S}_{CS} = \frac{1}{2\kappa_{10}^2} \int \frac{C_4 \wedge G \wedge \bar{G}}{4i\Im\tau}$$

This induces a RR tadpole for C_4 given by

$$N_{flux} = \frac{1}{2\kappa_{10}^2 \mu_3} \int H_R^{(3)} \wedge H_{NS}^{(3)} = m \times e$$

So we need D-branes and orientifold planes in order to cancel the flux RR-tadpole and the unbalanced flux vacuum energy!

$\Longrightarrow$ D9-branes with magnetic fluxes plus orientifold planes.

Example $Z_2 \times Z_2$ orientifold: 4 tadpole conditions

$$8 \sum_a \prod_I n_a^I + N_{flux} = 32\,, \quad 8 \sum_a N_a n_a^1 m_a^2 m_a^3 = \pm 32\,,$$
$$8 \sum_a N_a m_a^1 n_a^2 m_a^3 = -32\,, \quad 8 \sum_a N_a m_a^1 m_a^2 n_a^3 = -32$$

Total scalar potential:

$$\mathcal{V}_{total}(T_i, U_i, \tau) = \mathcal{V}_{flux}(U_i, \tau) + \mathcal{V}_{D9}(T_i, \tau) - \mathcal{V}_{O3,O7}(T_i, \tau)$$

IV) Type II Compactifications with D-branes and H-fluxes

Concrete example:

One can construct a $\mathcal{N} = 1$ supersymmetric $Z_2 \times Z_2$ orientifold model with supersymmetric D9-branes and supersymmetric 3-form fluxes:

(i) 3-form fluxes $\leftrightarrow$ complex structure moduli:

$$U^1 U^2 = -1 \,, \quad \tau = -U^3$$

(ii) 2 stacks of D9-branes:

$$1^{st} \;\; \text{stack}: \;\; (n^I, m^I) = \{(0,1),(1,-1),(1,-1)\}$$
$$2^{nd} \;\; \text{stack}: \;\; (n^I, m^I) = \{(1,0),(0,-1),(0,-1)\}$$

Kähler moduli:

$$T^2 T^3 = (4\pi^2 \alpha')^2$$

Gauge group: $G = U(4) \times U(4)$

Chiral fermions: $(4,4) + (4,\bar{4})$-representations (anomalous, canceled by inflow mechanism).

V) Conclusions

What did we learn: though very hard, it seems possible to derive the SM from brane constructions!

The challenge remains to construct realistic supersymmetric IBW models with the chiral spectrum of the MSSM and only a mild amount of vector-like matter.

Other interesting topics:

- Intersecting D6-branes can be lifted to M-theory on a G_2 manifold.

- Combine D-branes and flux compactications.

  (Blumenhagen, Taylor, D.L.; Cascales, Uranga; Behrndt, Cvetic)

- Computation of Yukawa couplings

  (Cremades, Ibanez, Marchesano; Cvetic, Papadimitriou)

- Dynamical supersymmetry breaking

  (Cvetic, Langacker, Wang)

V) Conclusions

- Flavor changing neutral currents

  (Abel, Masip, Santiago; Abel, Owen)

- Computation of open string amplitudes $\longrightarrow$ effective action (soft SUSY breaking terms, ...)

  (N. Bernard, R. Richter, S. Stieberger, D.L., work in progress)

- Study of tachyonic Higgs effect

  (F. Epple, work in progress)

V) Conclusions

Important question: Does it make at all sense to construct 4-dim. string vacua without knowing the dynamical selection process which determines the unique string ground state (if it exists)?

(Preliminary) answer: Probably Yes!

Statistics of string/M theory vacua:

(M. Douglas, hep-th/0303194)

Assume that we can construct the SM spectrum from strings in several ways, where the SM couplings for each model are statistically, i.e. uniformly distributed.

SM fills the following volume in the space of coupling constants (measured in natural units):

$$\delta V_{SM} \sim 10^{-238}$$

Therefore we need at least $\mathcal{O}(10^{238})$ brane/flux string vacua with SM spectrum in order to make the statistical statement that string theory contains the SM.

This seems to be possible!